UM-DAE Centre for Excellence in Basic Sciences
Course Structure and Syllabus
5-year Integrated MSc - Mathematics Stream

P: Physics, M: Mathematics, C: Chemistry, B: Biology, G: General, H: Humanity,
ME: Math Elective, MPr : Math Project

This is the EXISTING version of the course structure and syllabus for Mathematics for semesters V-X

(as of Nov 2016)

Semester V

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M501	Analysis III	[3+1]	4
M502	Algebra III	[3+1]	4
M503	Topology II	[3+1]	4
M504	Graph Theory	[3+1]	4
G501	Earth Science & Energy & Environmental Sciences	[3+1]	4
PM501	Numerical Analysis	[3+1]	4
		Lab Hours per Week	
PML501	Numerical Methods Laboratory	[4]	2
		Semester Credits	**26**

M501 : Analysis III (Measure Theory and Integration)

1. Sigma algebra of sets, measure spaces. Lebesgues outer measure on the Real line. Measurable set in the sense of Caratheodory. Translation invariance of Lebesgue measure. Existence of a non-Lebesgue measurable set. Cantor set- uncountable set with measure zero.
2. Measurable functions, types of convergence of measurable functions. The Lebesgue integral for simple functions, nonnegative measurable functions and Lebesgue integrable function, in general.
3. Convergence theorems- monotone and dominated convergence theorems.
4. Comparison of Riemann and Lebesgue integrals. Riemanns theorem on functions which are continuous almost everywhere.
5. The product measure and Fubinis theorem.
6. The L^p spaces and the norm topology. Inequalities of Hölder and Minkowski. Completeness of L^p and L^∞ spaces.

References

[1] H.L. Royden, Real Analysis, Pearson Education.

[2] G. DeBarra, Introduction to Measure Theory, Van Nostrand Reinhold.

[3] I. K. Rana, An Introduction to Measure and Integration, Narosa.

[4] H.S. Bear, A Primer on Lebesgue Integration, Academic press.

M502 : Algebra III (Galois Theory)

1. Prime and maximal ideals in a commutative ring and their elementary properties.
2. Field extensions, prime fields, characteristic of a field, algebraic field extensions, finite field extensions, splitting fields, algebraic closure, separable extensions, normal extensions,
3. Finite Galois extensions, Fundamental Theorem of Galois Theory.
4. Solvability by radicals.
5. Extensions of finite fields.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] D. S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[3] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Corporation, 1983.

[4] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

[5] R. Lidl and H. Niederreiter, Introduction to Finite Fields and Their Applications, Cambridge University Press, 1986.

M503 : Topology II

1. Review of some notions from Topology I. Basic Separation axions and first and second countability axioms. Examples.
2. Products and quotients. Tychonoff's theorem. Product of connected spaces is connected. Weak topology on X induced by a family of maps $f_\alpha : X \longrightarrow X_\alpha$ where each X_α is a topological space. The coherent topology on Y induced by a family of maps $g_\alpha : Y_\alpha \longrightarrow Y$ where Y_α are given topological spaces. Examples of quotients to illustrate the universal property such as embeddings of $\mathbb{R}P^2$ and the Klein's bottle in $\mathbb{R}^4$.
3. Completely regular spaces and its embeddings in a product of intervals. Compactification, Alexandroff and Stone-Cech compactifications.
4. Normal spaces and the theorems of Urysohn and Tietze. The metrization theorem of Urysohn.
5. Local compactness, local connectedness and local path-connectedness and their basic properties. If $q : X \longrightarrow Y$ is a quotient map and Z is locally compact Hausdorff space then $q \times \mathrm{id} : X \times Z \longrightarrow Y \times Z$ is also a quotient map.
6. Locally finite families of sets and Partitions of unity. Baire Category theorem for locally compact Hausdorff spaces.

References

[1] G. F. Simmons, *Topology and modern analysis*

[2] W. A. Sutherland, *Introduction to metric and topological spaces.*

[3] S. Willard, *General Topology*, Dover, New York.

M504 : Graph Theory

1. Fundamentals: Definitions and Examples.
2. Trees: Basic properties, Spanning trees and enumeration, Optimization and trees.
3. Digraphs, Eulerian graphs.
4. Matchings and Factors: Matchings in Bipartite Graphs, Applications and Algorithms, Matchings in General Graphs.
5. Connectivity and Paths: Cuts and connectivity, k-connected graphs, Network Flow Problems.
6. Graph Colouring: Vertex colouring and Upper Bounds, Structure of k-chromatic graphs, Enumerative aspects.
7. Line graphs and edge colouring, Hamiltonian cycles.
8. Planar graphs.
9. Additional topics: Ramsey Theory, Extremal Problems.

References

[1] J.A. Bondy and U.S.R Murty, Graph Theory with Applications.

[2] Douglas B. West, Introduction to Graph Theory.

[3] R.B. Bapat, Graphs and Matrices.

[4] Frank Harary, Graph Theory.

Semester VI

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M601	Analysis IV	[3+1]	4
M602	Algebra IV	[3+1]	4
M603	Differential Geometry & Applications	[3+1]	4
M604	Differential Equations & Dynamical Systems	[3+1]	4
M605	Probability Theory	[3+1]	4
PM601	Special Functions and Applications	[3+1]	4
H601	Ethics of Science and IPR	[2+0]	2
		Semester Credits	**26**

M601 : Analysis IV (Complex Analysis)

1. Complex numbers and Riemann sphere. Möbius transformations.
2. Analytic functions. Cauchy-Riemann conditions, harmonic functions, Elementary functions, Power series, Conformal mappings.
3. Contour integrals, Cauchy theorem for simply and multiply connected domains. Cauchy integral formula, Winding number.
4. Moreras theorem. Liouvilles theorem, Fundamental theorem of Algebra.
5. Zeros of an analytic function and Taylors theorem. Isolated singularities and residues, Laurent series, Evaluation of real integrals.
6. Zeros and Poles, Argument principle, Rouchs theorem.

References

[1] L.Ahlfors, Complex Analysis.

[2] R.V. Churchill and J. W. Brown, Complex Variables and Applications, International Student Edition,Mc-Graw Hill, 4th ed., 1984.

[3] B.R. Palka, An Introduction to Complex Function Theory, UTM Springer-Verlag, 1991.

M602 : Algebra IV (Rings and Modules: Some Structure Theory)

1. Recollection of modules, submodules, quotient modules, homomorphisms.
2. External and internal direct sums of modules.
3. Tensor product of modules over a commutative ring. Functorial properties of $\otimes$ and Hom.
4. Definitions and elementary properties of projective and injective modules over a commutative ring.
5. Structure of finitely generated modules over a PID. Applications to matrices and linear maps over a field: rational and Jordan canonical forms.
6. Simpl modules over a not necessarily commutative ring, modules of finite length, Jordan-Hölder Theorem, Schur's lemma.
7. (Optional, if time permits) Semisimple modules over a not necessarily commutative ring, Wedderburn Structure Theorem for semisimple rings.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[3] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Corporation, 1983.

[4] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

M603 : Differential Geometry & Applications

1. Curvature of curves in $\mathbb{E}^n$: Parametrized Curves, Existence of Arc length parametrization, Curvature of plane curves, Frennet-Serret theory of (arc-length parametrized) curves in $\mathbb{E}^3$, Curvature of (arc-length parametrized) curves in $\mathbb{E}^n$, Curvature theory for parametrized curves in $\mathbb{E}^n$. Significance of the sign of curvature, Rigidity of curves in $\mathbb{E}^n$.

2. Euler's Theory of curves on Surfaces : Surface patches and local coordinates, Examples of surfaces in $\mathbb{E}^3$, curves on a surface, tangents to the surface at a point, Vector fields along curves, Parallel vector fields, vector fields on surfaces, normal vector fields, the First Fundamental form, Normal curvature of curves on a surface, Geodesics, geodesic Curvature, Christoffel symbols, Gauss' formula, Principal Curvatures, Euler's theorem.

3. Gauss' theory of Curvature of Surfaces : The Second Fundamental Form, Weingarten map and the Shape operator, Gaussian Curvature, Gauss' *Theorema Egregium*, Guass-Codazzi equations, Computation of First/Second fundamental form, curvature etc. for surfaces of revolution and other examples.

4. More Surface theory : Isoperimetric Inequality, Mean Curvature and Minimal Surfaces (introduction), surfaces of constant curvature, Geodesic coordinates, Notion of orientation, examples of non-orientable surfaces, Euler characteristic, statement of Gauss-Bonnet Theorem.

5. Modern Perspective on Surfaces : Tangent planes, Parallel Transport, Affine Connections, Riemannian metrics on surfaces.

References

[1] Elementary Differential Geometry : Andrew Pressley, Springer Undergraduate Mathematics Series.

[2] Elementary Differential Geometry : J. Thorpe, Elsevier.

[3] Differential Geometry of Curves and Surfaces : M. do Carmo.

[4] Elements of Differential Geometry : R. Millman & G. Parker.

M604 : Differential Equations & Dynamical Systems

1. Basic existence and uniqueness of systems of ordinary differential equations satisfying the Lipschitz' condition. Examples illustrating non-uniqueness when Lipschitz or other relevant conditions are dropped. Gronwall's lemma and its applications to continuity of the solutions with respect to initial conditions. Smooth dependence on initial conditions and the variational equation. Maximal interval of existence and global solutions. Proof that if (a, b) is the maximal interval of existence and $a < \infty$ then the graph of the solution must exit every compact subset of the domain on the differential equation.

2. Linear systems and fundamental systems of solutions. Wronskians and its basic properties. The Abel Liouville formula. The dimensionality of the space of solutions. Fundamental matrix. The method of variation of parameters.

3. Linear systems with constant coefficients and the structure of the solutions. Matrix exponentials and methods for computing them. Solving the in-homogeneous system. The Laplace transform and its applications.
4. Second order scalar linear differential equations. The Sturm comparison and separation theorems and regular Sturm-Liouville problems.
5. Series solutions of ordinary differential equations and a detailed analytic study of the differential equations of Bessel and Legendre.
6. Dynamical systems and basic notions of dynamical systems such as flows, rectification theorem, rest-points and its stability. Liouville's theorem on the preservation of phase volume. First integrals and their applications.

References

[1] R. Courant and D. Hilbert, Methods of Mathematical Physics, Volume - I

[2] W. Hurewicz, *Lectures on ordinary differential equations*, Dover, New York.

[3] G. F. Simmons, *Differential equations with applications and historical notes,* McGraw Hill.

M605 : Probability Theory

1. Probability as a measure, Probability space, conditional probability, independence of events, Bayes formula. Random variables, distribution functions, expected value and variance. Standard Probability distributions: Binomial, Poisson and Normal distribution.
2. Borel-Cantelli lemmas, zero-one laws. Sequences of random variables, convergence theorems, various modes of convergence. Weak law and the strong law of large numbers.
3. Central limit theorem: DeMoivre-Laplace theorem, weak convergence, characteristic functions, inversion formula, moment generating function.
4. Random walks, Markov Chains, Recurrence and Transience.
5. Conditional Expectation, Martingales.

References

[1] Marek Capinski and Tomasz Zastawniak, Probability through Problems, Springer, Indian Reprint 2008.

[2] P. Billingsley, Probability and Measure, 3rd ed., John Wiley & Sons, New York, 1995.

[3] J. Rosenthal, A First Look at Rigorous Probability, World Scientific, Singapore, 2000.

[4] A.N. Shiryayev, Probability, 2nd ed., Springer, New York, 1995.

[5] K.L. Chung, A Course in Probability Theory, Academic Press, New York, 1974.

PM601 : Special Functions and Applications

Module 1: Special Functions: Linear differential operators, self adjoint differential operators, self adjoint second order differential operators, Strurm Liouville eigenvalue problem. Power series solutions to differential equations. Notion of generating functions. Definition of Legendre polynomials, Rodrigues' formula, Legendre differential equation, integral properties of Legendre polynomials, Legendre functions, recurrence formula, Legendre function of 2nd kind, its properties. Associated Legendre functions of first and second kinds, and their properties, Spherical harmonics and their properties. Hypergeometric series. Differential equation satisfied by hypergeometric series. Hypergeometric functions. Legendre functions as a particular case of hypergeometric function. Beta functons and Gamma functions. Bessel Functions: The Bessel differential equation, Bessel's equation as a case of hypergeometric equation, general solution of the Bessel's equation, recurrence formulae, Bessel functions of first and second kind, roots of Bessel functions, normalisation of Bessel functions, Hankel functions. Asymptotic and threshold limits of the Bessel's functions. A brief discussion on Hermite, Laguerre, Tschebyscheff polynomials.

Module 2: Green's Functions: The concept of a Green's function through a one dimensional boundary value problem: forced, transverse vibrations of a taut string of a given length, properties of the Green's function thus obtained. Review of the self adjoint second order differential operators. Non homogeneous problems. Green's functions and second order differential operators. Green's functions and the adjoint operator. Spectral representation and Green's functions. A brief overview of integral equations: classification, method of successive approximations, symmetric integral equations, equivalence of integral and differential equations. Green's functions in two and three dimensional spaces. Partial differential operators, 'fundamental' solutions of boundary value problems (a brief discussion), self adjoint elliptic equations: Dirichlet and Neumann problems, parabolic and hyperbolic equations, calculation of particular Green's functions: the method of images, boundary value problems in electrodynamics etc.

References

[1] Mathematics for Physicists: Philippe Dennery and Andre Krzywicki

[2] Mathematics for Quantum Mechanics: J. D. Jackson

[3] Mathematical Methods for Physicists: George Arfken and Hans Weber.

[4] A Course on Modern Analysis: E. T. Whittaker

[5] Green's Functions: G. F. Roach

[6] Classical Electrodynamics: J. D. Jackson

Semester VII

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M701	Functional Analysis	[3+1]	4
M702	Commutative Algebra	[3+1]	4
M703	Stochastic Analysis	[3+1]	4
M704	Partial Differential Equations	[3+1]	4
M705	Representation Theory of Finite Groups	[3+1]	4
MPr701	Project		4
		Semester Credits	**24**

M701 : Functional Analysis

1. Normed linear spaces. Riesz lemma. Heine-Borel theorem. Continuity of linear maps.
2. Hahn-Banach extension and separation theorems.
3. Banach spaces. Subspaces, product spaces and quotient spaces. Standard examples of Banach spaces like ℓ^p, L^p, $C([0,1])$ etc.
4. Uniform boundedness principle. Closed graph theorem. Open mapping theorem. Bounded inverse theorem.
5. Spectrum of a bounded operator. Eigenspectrum. Gelfand-Mazur theorem and spectral radius formula.
6. Dual spaces. Transpose of a bounded linear map. Standard examples.
7. Hilbert spaces. Bessel inequality, Riesz-Schauder theorem, Fourier expansion, Parseval's formula.
8. In the framework of a Hilbert space: Projection theorem. Riesz representation theorem. Uniqueness of Hahn-Banach extension.

References

[1] J.B. Conway, A course in Functional Analysis, Springer-Verlag, Berlin, 1985.

[2] G. Goffman and G. Pedrick, First course in functional analysis, Prentice-Hall, 1974.

[3] E. Kreyszig, Introductory Functional Analysis with applications, John Wiley & Sons, NY, 1978.

[4] B.V. Limaye, Functional Analysis, 2nd ed., New Age International, New Delhi, 1996.

[5] A. Taylor and D. Lay, Introduction to functional analysis, Wiley, New York, 1980.

M702 : Commutative Algebra

1. Prime and maximal ideals in a commutative ring, nil and Jacobson radicals, Nakayamas lemma, local rings.
2. Rings and modules of fractions, correspondence between prime ideals, localization.
3. Modules of finite length, Noetherian and Artinian modules.
4. Primary decomposition in a Noetherian module, associated primes, support of a module.
5. Graded rings and modules, Artin-Rees, Krull-intersection,
6. Hilbert-Samuel function of a local ring, dimension theory, principal ideal theorem.
7. Integral extensions, Noethers normalization lemma, Hilberts Nullstellensatz (algebraic and geometric versions).

References

[1] M.F Atiyah and I.G MacDonald, Introduction to Commutative Algebra, Addison-Wesley, 1969.

[2] D. Eisenbud, Commutative Algebra with a view toward algebraic geometry, Springer-Verlag, Berlin, 2003.

[3] H. Matsumura, Commutative ring theory, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press, Cambridge, 1980.

[4] S. Raghavan, B. Singh and R. Sridharan, Homological methods in commutative algebra, TIFR Math. Pamphlet No.5, Oxford, 1975.

[5] B. Singh, Basic Commutative Algebra, World Scientific, 2011.

703 : Stochastic Analysis

1. Preliminaries :
   (i) Martingales and properties.
   (ii) Brownian Motion- definition and construction, Markov property, stopping times, strong Markov property zeros of one dimensional Brownian motion.
   (iii) Reflection principle, hitting times, higher dimensional Brownian Motion, recurrence and transience, occupation times, exit times, change of time, Levys theorem.
2. Stochastic Calculus :
   (i) Predictable processes, continuous local martingales, variance and covariance processes.
   (ii) Integration with respect to bounded martingales and local martingales, Kunita Watanabe inequality, Ito s formula, stochastic integral, change of variables.
   (iii) Stochastic differential equations, weak solutions, Change of measure , Change of time, Girsanovs theorem.

References

[1] Richard Durrett, Stochastc Calculus A Practical Introduction, CRC Press 1996.

[2] Karatzas I. and Steven Shreve, Brownian Motion and Stochastic Calculus, Springer.

[3] Oksendal Bernt, Stochastic Differential Equations, Springer.

[4] J.Michael Steele, Stochastic Calculus and Financial Applications, Springer, 2000

M704 : Partial Differential Equations

1. Generalities on the origins of partial differential equations. Generalities on the Cauchy problem for a scalar linear equation of arbitrary order. The concept of characteristics. The Cauchy-Kowalevskya theorem and the Holmgren's uniqueness theorem. The fundamental equations of mathematical physics as paradigms for the study of Elliptic, Hyperbolic and Parabolic equations.

2. Quasilinear first order scalar partial differential equations and the method of characteristics. Detailed discussion of the inviscid Burger's equation illustrating the formation of discontinuities in finite time. The fully nonlinear scalar equation and Eikonal equation. The Hamilton-Jacobi equation.

3. Detailed analysis of the Laplace and Poisson's equations. Green's function for the Laplacian and its basic properties. Integral representation of solutions and its consequences such as the analyticity of solutions. The mean value property for harmonic functions and maximum principles. Harnack inequality.

4. The wave equation and the Cauchy problem for the wave equation. The Euler-Poisson-Darboux equation and integral representation for the wave equation in dimensions two and three. Properties of solutions such as finite speed of propagation. Domain of dependence and domain of influence.
5. The Cauchy problem for the heat equation and the integral representation for the solutions of the Cauchy problem for Cauchy data satisfying suitable growth restrictions. Infinite speed of propagation of signals. Example of non-uniqueness.
6. Fourier methods for solving initial boundary value problems.

References

[1] R. Courant and D. Hilbert, Methods of Mathematical Physics, Volume - II

[2] R. C. McOwen, *Partial differential equations*, Pearson Education, 2004.

M705 : Representation Theory of Finite Groups

1. Recollection of left and right modules, direct sums, tensor products.
2. Semi-simplicity of rings and modules, Schur′s lemma, Maschke′s Theorem, Wedderburn′s Structure Theorem.
3. The group algebra.
4. Representations of a finite group over a field, induced representations, characters, orthogonality relations.
5. Representations of some special groups.
6. Burnside's $p^a q^b$ theorem.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.

[3] D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[4] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Corporation, 1983.

[5] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

[6] J.P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.

Semester VIII

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M801	Fourier Analysis	[3+1]	4
M802	Algebraic Number Theory	[3+1]	4
M803	Algebraic Topology	[3+1]	4
M804	Differential Topology	[3+1]	4
M805	Computational Mathematics III	[3+1]	4
MPr801	Project		6
		Semester Credits	**26**

M801 : Fourier Analysis

1. Fourier series. Discussion of convergence of Fourier series.
2. Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Summability, Fejer's theorem, Dirichlets theorem, Poisson Kernel and summability kernels. Example of a continuous function with divergent Fourier series.
3. Summability of Fourier series for functions in L^1, L^2 and L^p spaces. Fourier-transforms of integrable functions. Basic properties of Fourier transforms, Poisson summation formula, Hausdorff-Young inequality, Riesz-Thorin Interpolation theorem.
4. Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians, Plancheral theorem, Paley-Weiner theorem.
5. Distributions and Fourier Transforms: Calculus of Distributions, Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs.

References

[1] Y. Katznelson, Introduction to Harmonic Analysis, Dover.

[2] R. E. Edwards, Fourier Series, Academic Press.

[3] E. M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, Princeton 2003.

[4] W. Rudin, Fourier Analysis on groups, Interscience.

M802 : Algebraic Number Theory

1. Field extensions and examples of field extensions of rational numbers, real numbers and complex numbers. Monic polynomials, Integral extensions, Minimal polynomial, Characteristic polynomial.
2. Integral closure and examples of rings which are integrally closed. Examples of rings which are not integrally closed. The ring of integers. The ring of Gaussian integers. Quadratic extensions and description of the ring of integers in quadratic number fields.
3. Units in quadratic number fields and relations to continued fractions.
4. Noetherian rings, Rings of dimension one. Dedekind domains. Norms and traces. Derive formulae relating norms and traces for towers of field extensions.
5. Discriminant and calculations of the discriminant in the special context of quadratic number fields. Different and it's applications.

6. Cyclotomic extensions and calculation of the discriminant in this case. Factorization of ideals into prime ideals and its relation to the discriminant.

7. Ramification theory, residual degree and its relation to the degree of the extension. Ramified primes in quadratic number fields.

8. Ideal class group. Geometric ideas involving volumes. Minkowski's theorem and its application to proving finiteness of the ideal class group.

9. Real and complex embeddings. Structure of finitely generated abelian groups. Dirichlet's Unit Theorem and the rank of the group of units. Discrete valuation rings, Local fields.

References

[1] Janusz, Algebraic Number Fields.

[2] Neukirch, Algebraic Number Theory.

[3] Marcus, Number Fields.

M803 : Algebraic Topology

1. Review of quotient spaces and its universal properties. Examples on $\mathbb{R}P^n$, Klein's bottle, Möbius band, $\mathbb{C}P^n$, $SO(n, \mathbb{R})$. Connectedness and path connectedness of spaces such as $SO(n, \mathbb{R})$ and other similar examples. Topological groups and their basic properties. Proof that if H is a connected subgroup such that G/H is also connected (as a topological space) then G is connected. Quaternions, S^3 and $SO(3, \mathbb{R})$. Connected, locally path connected space is path connected.

2. Paths and homotopies of paths. The fundamental group and its basic properties. The fundamental group of a topological group is abelian. Homotopy of maps, retraction and deformation retraction. The fundamental group of a product. The fundamental group of the circle. Brouwer's fixed point theorem. Degree of a map. Applications such as the fundamental theorem of algebra, Borsuk-Ulam theorem and the Perron Frobenius theorem.

3. Covering spaces and its basic properties. Examples such as the real line as a covering space of a circle, the double cover $\eta : S^n \longrightarrow \mathbb{R}P^n$, the double cover $\eta : S^3 \longrightarrow SO(3, \mathbb{R})$. Relationship to the fundamental group. Lifting criterion and Deck transformations. Equivalence of covering spaces. Universal covering spaces. Regular coverings and its various equivalent formulations such as the transitivity of the action of the Deck group. The Galois theory of covering spaces.

4. Orbit spaces. Fundamental group of the Klein's bottle and torus. Relation between covering spaces and Orientation of smooth manifolds. Non orientabilty of $\mathbb{R}P^2$ illustrated via covering spaces.

5. Free groups and its basic properties, free products with amalgamations. Concept of push outs in the context of topological spaces and groups. Seifert Van Kampen theorem and its applications. Basic notions of knot theory such as the group of a knot. Wirtinger's algorithm for calculating the group of a knot illustrated with simple examples.

References

[1] E. L. Lima, *Fundamental groups and covering spaces*, A. K. Peters, 2003.

[2] W. Massey, *Introduction to algebraic topology.* Springer Verlag.

M804 : Differential Topology

1. Differentiable functions on $\mathbb{R}^n$: Review of differentiable functions $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$, Implicit and Inverse function theorems, Immersions and Submersions, critical points, critical and regular values.

2. Manifolds : Level sets, sub-manifolds of $\mathbb{R}^n$, immersed and embedded sub-manifolds, tangent spaces, differentiable functions between sub-manifolds of $\mathbb{R}^n$, abstract differential manifolds and tangent spaces.

3. Differentiable functions on Manifolds : Differentiable functions $f : M \longrightarrow N$, critical points, Sard's theorem, non-degenerate critical points, Morse Lemma, Manifolds with boundary, Brouwer fixed point theorem, *mod 2* degree of a mapping.

4. Transversality : Orientation of Manifolds, oriented intersection number, Brouwer degree, transverse intersections.

5. Integration on Manifolds : Vector field and Differential forms, integration of forms, Stokes' theorem, exact and closed forms, Poincar Lemma, Introduction to de Rham theory.

References

[1] Topology from a Differentiable Viewpoint : J. Milnor.

[2] Differential Topology : V. Guellemin & A. Pollack.

[3] Differential Topology : M. Hirsch.

M805 : Computational Mathematics III

1. Differential Geometry of curves and surfaces using Mathematica.
2. Exploring Differential Equation and Dynamical System using XPPAUT or some other specialized sofware.
3. Design of Experiments and Statistics Quality control using R.
4. Project/Math Modeling problem using any Mathematical Software and developing Mathematica packages for various specific methods.
5. Exploring solutions of Partial Differential equations using Mathematica. Developing programmes to solve problems numerically.
6. Exploring basic Notions of Commutative algebra using Sage/Singular /Kash etc.
7. Advanced notion of optimization techniques using Mathematica.
8. Project/Math Modeling problem using any Mathematical Software and developing Mathematica packages for various specific methods.

References

[1] Alfred Gray, Elsa Abbena, Simon Salamon, Modern Differential Geometry of Curves and Surfaces with Mathematica, Third Edition.

Semester IX

Subject Code	**Subject**	**Contact Hours per Week** Theory + Tutorials	**Credits**
MPr901	Project		24
		Semester Credits	**24**

Semester X

Subject Code	**Subject**	**Contact Hours per Week** Theory + Tutorials	**Credits**
ME100*	Elective *	[3+1]	4

Minimum credits required: 240

List of electives appears on the next sheet

Electives

Electives taught in previous semesters

1. Advanced Commutative Algebra & Applications.
2. Advanced Differential Topology.
3. Advanced Numerical Techniques.
4. Analytic Number Theory.
5. Coding Theory & Cryptography.
6. Combinatorics & Enumeration.
7. Financial Mathematics.
8. Fractals & Applications.
9. Geometry of Numbers.
10. Graph Theory.
11. Introduction to Ergodic Theory.
12. Lie Groups & Geometry
13. Quantum Computing.
14. Topics in Algebraic Geometry.

Other possible electives

1. Advanced Algebraic Topology & Applications.
2. Advanced Differential Geometry & Applications.
3. Algebraic curves.
4. Class field theory.
5. Combinatorial Design Theory.
6. Econometrics.
7. Elliptic curves.
8. Finite Fields & Applications.
9. Fluid Mechanics.
10. Homological Algebra & Applications.
11. Industrial Mathematics.
12. Introduction to algebraic groups.
13. Mathematical Applications to Engineering.
14. Mathematics & Nano Technology.
15. Modular forms.
16. Operator Theory.
17. Perturbation Theory.
18. Wavelet Analysis & Applications.