मौलिक विज्ञान प्रकर्ष केन्द्र

UM-DAE Centre for Excellence in Basic Sciences

WHAT ARE BERNOULLI NUMBERS?

Colloquium by

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ABSTRACT

The Bernoulli Numbers is the sequence of rational numbers $\{B_n\}_{0\leq n<\infty}$ defined by the identity $\frac{x}{e^x-1} = \sum_{0\leq n<\infty} B_n \frac{x^n}{n!}$ valid for $\{x \in \mathbb{R} | |x| < 1|\}$.
These numbers crop up in diverse contexts in Mathematics. They are named after the Swiss mathematician Jakob Bernoulli (1654-1705) who introduced them in the context of giving a formula for the sum $S_{k(n)}$ of $\sum_{1\leq r\leq n} r^k$ for the sums of k^{th} powers of integers up to n for an integer $k \geq 0$. In this talk I will give a proof of Bernoulli's formula for $S_{k(n)}$.
When one considers these sums for negative k, one cannot provide a closed formula for the sum $S_{k(n)}$. However the infinite series $\sum_{1\leq n<\infty} n^k$ converges for $k \geq 2$ and is denoted $\zeta(k)$.

Leonard Euler (1707-1783) showed that for even integers $2r \geq 2$, $\zeta(2r) = (-1)^{(r+1)} B_{2n} \frac{(2\pi)^{2n}}{2(2n)!}$ and I will indicate a proof of this as well.
Bernoulli numbers spring up in other contexts as well, notably in differential topology.